Investigation of the golf clubhead-ball impact

Research Question: What is the relationship between the dynamic loft of the golf club and the spin, launch angle, and speed of the golf ball?

Physics Extended Essay

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Introduction

A main focus in the game of golf is controlling the spin, launch angle, and speed of golf shots, whether to stop the ball on the green quickly or to achieve optimal driving distance. As a passionate golfer for five years, I have been taught many techniques by professional coaches to control these factors on different types of shots, mostly by manipulating the dynamic loft of the club. This led me to the research question:

What is the relationship between the dynamic loft of the golf club and the spin, launch angle, and speed of the golf ball?

Through this investigation, I hope to gain a more comprehensive understanding of the physics behind the golf clubhead-ball impact and thus learn to better control my golf shots. This research question will be answered by attempting to theoretically model the golf clubhead-ball impact phenomenon with physics concepts before evaluating this model against experimental data to determine its accuracy.

Background Information

Useful definitions (Hahn, 2021)

Figure 1. The clubhead-ball impact at maximum compression (Down The Middle, 2020)

Dynamic Loft: The vertical angle of the clubface at the time of maximum compression

Spin Rate: The ball's angular velocity immediately after impact about its axis of rotation

Launch Angle: Angle of the ball's velocity to the horizontal immediately after impact

Ball speed: The magnitude of the velocity of the ball immediately after impact

Attack Angle: Angle of the club's velocity to the horizontal immediately before impact

Background Theory

The golf club consists of a metal clubhead attached to a steel shaft. Modern golf balls consist of a rubber core and a urethane cover with indents called dimples. The clubhead-ball impact is an extremely violent one which occurs over a period less than 0.5 ms during which the ball accelerates up to speeds exceeding 60 $m\,s^{-1}$ (Penner, 2001) and is compressed significantly (**Figure 1**).

Existing Models of the golf clubhead-ball impact

- Several studies have used finite element analysis (FEA) to successfully model the golf clubhead-ball impact (United States Golf Association, 2006; Keatley, 2021; Iwatsubo et al., 1998; Maw et al. 1976). The resulting models described this impact behaviour with detail and accuracy. It considers the dynamic contact forces and torques throughout the impact and the ball's deformation by numerically solving partial differential equations to compute the system's behaviour.
- Penner (2001) and United States Golf Association (2006) attempted to model the impact with rigid body mechanics. The concepts of conservation of linear and angular momenta and coefficient of restitution are utilised.

Although FEA better captures the impact behaviour, it is not practical because operating FEA software is beyond my abilities. USGA (2006) revealed that the golf ball's behaviour can still be well described by the dynamics of a rigid sphere. Additionally, the focus of this investigation is on the final spin, launch angle and speed of the ball rather than capturing the impact behaviour. So this investigation will consider both the ball and clubhead as rigid objects.

Impact behaviour

The effect of the club shaft on the clubhead during the collision can be ignored because the force exerted by the shaft is negligible compared to the force of impact (Cochran & Stobbs, 1968). Their study tested a modified golf club where the clubhead is hinged on the shaft, flash photographs and the club's performance showed no significant difference to normal golf clubs. Therefore, the golf clubhead-ball impact can be modelled as the free-body oblique impact between a sphere and an inclined plane.

Previous studies and video evidence have shown that two conditions can be distinguished for the golf clubhead-ball impact: sliding and rolling (Maw et al., 1976; Penner, 2001; Down the Middle, 2020). After contact with the club, the ball will begin to slide up along the clubface due to its mass and inertia. Frictional force between the clubface and the ball will oppose this motion, reducing the relative velocity between the two surfaces at the point of contact. If this velocity reaches zero, the ball will begin to roll up the clubface and friction will instantaneously drop to zero since there is no relative motion between the two surfaces. Once established, this rolling motion will persist until the end of the impact (Maw et al., 1976; Penner, 2001).

Clubhead motion and impact conditions

In a typical golf swing, the club travels at an angle of depression of around 0-4 degrees. During impact, the club travels along an approximate arc with its radius being the combined length of the club and the player's arms. The clubhead-ball system moves across approximately 0.02 m during impact. This is negligible compared to the radius of the arc, hence, the club's motion during impact is approximately linear.

During impact, the clubface is not always perpendicular to the target line and the clubhead does not always travel along the target line despite players' efforts to achieve this 'perfect' shot. Similarly, the point of contact between the club and the ball is not always at the centre of the clubface (**Figure 2b**). These inconsistencies result in forces and torques in directions perpendicular to the target line (Iwatsubo et al., 1998). This means that the spin axis and the ball's velocity are not always normal and parallel to the target line respectively. However, for shots which players intend to hit straight, they only deviate by angles less than 1 degree. Hence, these minor inconsistencies lead to negligible difference in the spin rate and ball speed when compared to a 'perfect' shot:

$$backspin = \cos(\theta) \times spin \quad ball's\ velocity\ along\ target\ line = \cos(\theta) \times ball\ speed$$

$$\lim_{\theta \to 0} cos(\theta) = 1\ ; \ \ small\ angle\ approximation$$

$$backspin = spin \quad ball's\ velocity\ along\ target\ line = ball\ speed$$

Therefore, it is reasonable to assume that the clubhead travels along the target line, the clubface is perpendicular to the target line, and the location of impact is at the centre of the clubface as shown in **Figures 2a.** and **2b.**

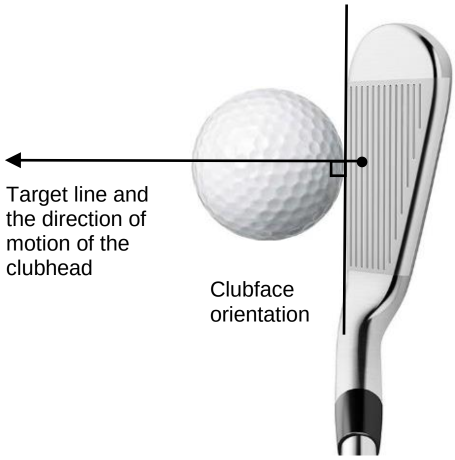


Figure 2a. Top-down view of the clubhead (TaylorMade Golf, n.d.) and ball (Shutterstock, n.d.) demonstrating clubhead motion and clubface orientation

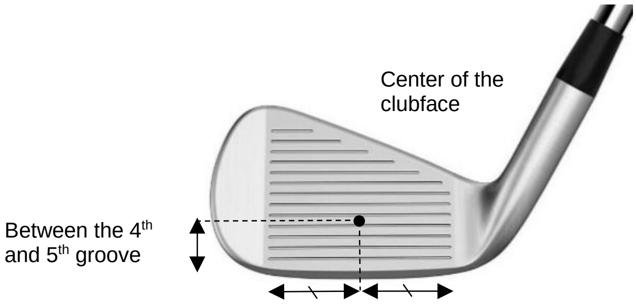


Figure 2b. Front on view of the clubhead (TaylorMade Golf, n.d.) demonstrating the assumed location of impact

Physical Properties

It is impossible to accurately determine the position of the centre of mass of the clubhead as the geometry, material, and mass distribution are not disclosed by their manufacturers. This investigation will assume that the centre of mass of the clubhead lies on the line connecting that of the ball and the centre point of impact. This line is fairly close to the geometric centre of the clubhead which is likely to be very close to its true centre of mass. This assumption is necessary because I have no means of obtaining either the moment of inertia or the angular velocity of the clubhead after the impact. Under this assumption, the clubhead should not have any rotational motion after the impact as the impact force acts perpendicular to the clubface through its centre of mass. These assumptions eliminate any forces, torques, and motion in directions perpendicular to the target line. Consequently, the clubhead-ball impact can be considered on a 2-D plane on which lies the line of impact with reference to axes normal and parallel to the clubface as shown in **Figure 3**.

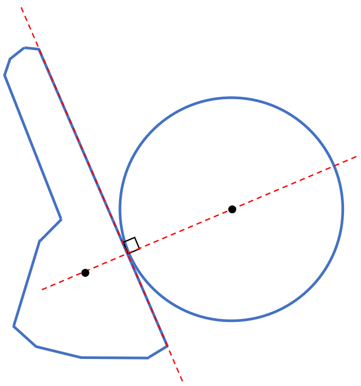


Figure 3. Side view of the clubhead-ball impact. The club is represented by a silhouette of its side profile. Red axes are axes normal and parallel to the clubface. Black dots represent centres of mass. (Diagram not to scale)

The golf ball will be modelled as a homogeneous rigid sphere with a mass of 0.04593 kg and a diameter of 0.04267 m as these are limitations set by the USGA (USGA & R&A, 2019). The golf ball is approximated to be a sphere because the effect of dimples cannot be quantitatively determined.

The theoretical model

Assumptions

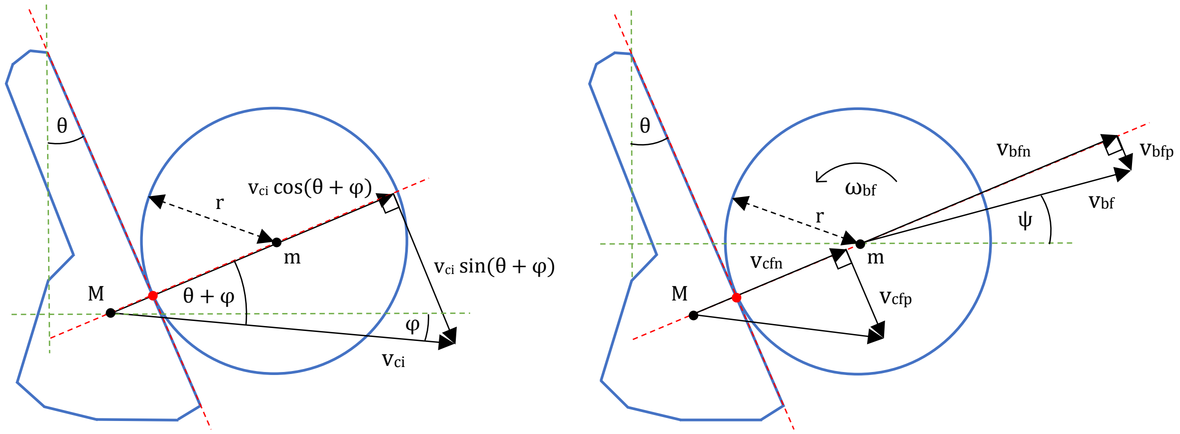
Although these simplifications come at the cost of the conclusion's accuracy, no quantitative analysis can be done without them. The assumptions discussed in background theory are summarized:

1. Both the club and the ball are rigid bodies.
2. The shaft does not affect the clubhead-ball system during impact.
3. The ball enters and remains in rolling motion.
4. The clubhead's initial motion is linear.
5. The clubhead travels along the target line, the clubface is perpendicular to the target line, and the location of impact is at the centre of the clubface.
6. The centre of mass of the clubhead lies on the line connecting that of the ball and the centre point of impact.
7. Any effects of the dimples are negligible.

List of variables and constants

Mass of clubhead [kg]	M
Mass of golf ball [kg]	m
Radius of golf ball [m]	r
Final Angular Velocity of golf ball [rad s^{-1}]	ω_{bf}
Spin Rate of golf ball [revolutions per minute (rpm)]	S
Attack angle of clubhead [degrees]	φ
Dynamic Loft of clubhead [degrees]	θ
Velocity of the clubhead before impact [m s^{-1}] (magnitude)	v_{ci}
Velocity of the ball after impact [m s^{-1}] (magnitude, normal, and parallel component)	v_{bf}, v_{bfn}, v_{bfp}
Velocity of the clubhead after impact [m s^{-1}] (normal and parallel component	v_{cfn}, v_{cfp}
Moment of inertia of the golf ball about an axis through its centre of mass	I
Coefficient of Restitution along the normal axis	e
Launch angle of the ball [degrees]	ψ

Conservation of linear momentum



Figures 4a. and 4b. Side view of the system before and after the impact respectively. Red axes are normal and parallel to the clubface, green axes are horizontal and vertical. Velocities of the club and the ball are represented with their components normal and parallel to the clubface.

During impact, the total linear momentum of the clubhead-ball system is conserved. Conservation of momentum in directions normal and parallel to the clubface give equations (1) and (2) respectively:

$$Mv_{ci}\cos(\theta + \varphi) = Mv_{cfn} + mv_{bfn} \tag{1}$$

$$Mv_{ci}\sin(\theta + \varphi) = Mv_{cfp} + mv_{bfp} \tag{2}$$

where v_{cfn} and v_{cfp} are components of the club's velocity after impact, and v_{bfn} and v_{bfp} are components of the ball's velocity after impact.

Conservation of angular momentum

The initial angular momentum of the system is zero because the ball is stationary and the club moves with linear motion. After impact, the ball has anti-clockwise angular momentum of $I\omega_{bf}$ due to its rotation. However, the system also has angular momentum in the clockwise direction because the point of contact is instantaneously at rest and the centre of mass of the ball has tangential velocity v_{bfp} relative to the point of contact in the clockwise direction. This results in rotation about the point of contact and therefore clockwise angular momentum of $mv_{bfp}r$. Conservation of angular momentum requires these 2 components to be equal and opposite. Therefore, taking the conservation of angular momentum about the point of contact gives:

$$I\omega_{bf} + mv_{bfp}r = 0 \tag{3}$$

where

$$I = \frac{2}{5}mr^2 \tag{4}$$

for the golf ball, a homogenous sphere.

Coefficient of restitution

A considerable amount of energy is lost to sound and material wear and tear during the clubhead-ball impact. Various studies have demonstrated that the modern golf ball is visco-hyperelastic; it deviates from Hooke's law. Specifically, the force acting on the ball at the contact point during the compression phase of the impact is greater than that acting during the restitution phase. (Cermik et al., 2017; Penner, 2001; Mase, 2004). This also results in a loss of kinetic energy to heat within the ball.

The coefficient of restitution is the ratio between the final and initial relative velocities of two objects after an interaction and accounts for this energy loss. The coefficient of restitution of the clubhead-ball impact in the normal direction is determined by:

$$e = \frac{v_{bfn} - v_{cfn}}{v_{ci}\cos(\theta + \varphi)} \tag{5}$$

The coefficient of restitution in the direction normal to the clubface, based on experimental data, can be given by (Lieberman & Johnson, 1994; Penner, 2001):

$$e = 0.86 - 0.0029v_{ci}\cos(\theta + \varphi) \tag{6}$$

Although Lieberman & Johnson's (1994) study is quite dated, the above relationship is in agreement with more recent studies conducted using modern golf balls (Haron & Ismail, 2012, Mase, 2004; USGA, 2006). All studies demonstrated a negative relationship between the club's velocity and the coefficient of restitution and yielded values comparable to numbers produced by equation (6). Hence, equation (6) will be used to obtain the coefficient of restitution in this investigation.

Slipping and Rolling motion

The ball will only enter rolling motion if there is sufficient friction force. It was found that the spin rate of the golf ball was independent of the clubface for typical loft angles of drivers (Chou et al., 1994). This suggests that the ball enters and remains in pure rolling motion until the end of the impact (Cochran & Stobbs, 1968). Since this investigation will not be able to determine forces normal to the clubface and the coefficient of friction, it will be assumed that impact with every dynamic loft will end in pure rolling motion. This assumption gives the relationship:

$$v_{cfp} - v_{bfp} = \omega_{bf} r \qquad (7)$$

If the ball is in pure rolling motion as it leaves the clubface, their relative velocity in the parallel direction at the point of contact (**Figure 4b**) must be zero. The relative velocity between the 2 surfaces at the contact point, $v_{cfp} - v_{bfp}$, therefore needs to be made up by the tangential velocity of the ball's rotation, $\omega_{bf} r$, for the ball to be in pure rolling motion.

Derivation of governing equations

Solving the above system of equations allows the spin rate, ball speed, and launch angle of a shot to be determined given the clubhead speed, dynamic loft, and attack angle:

Rearranging equation (5) for v_{cfn} gives:

$$v_{cfn} = v_{bfn} - ev_{ci}\cos(\theta + \varphi)$$

Substituting into equation (1) to solve for v_{bfn}:

$$Mv_{ci}\cos(\theta + \varphi) = M(v_{bfn} - ev_{ci}\cos(\theta + \varphi)) + mv_{bfn}$$

Expanding and rearranging:

$$Mv_{bfn} + mv_{bfn} = Mv_{ci}\cos(\theta + \varphi) + Mev_{ci}\cos(\theta + \varphi)$$

Factorizing and dividing:

$$v_{bfn} = \frac{Mv_{ci}\cos(\theta + \varphi) + Mev_{ci}\cos(\theta + \varphi)}{M + m}$$

Factorizing and dividing by M gives:

$$v_{bfn} = \frac{v_{ci}\cos(\theta + \varphi)(1 + e)}{1 + \frac{m}{M}} \quad (8)$$

Rearranging equation (3) for ω_{bf} and equation (7) for v_{cfp} gives:

$$\omega_{bf} = \frac{-mv_{bfp}r}{I}$$

and

$$v_{cfp} = v_{bfp} + \omega_{bf}r$$

Substituting ω_{bf} into equation (7) gives:

$$v_{cfp} = v_{bfp} + \frac{mv_{bfp}r^2}{I}$$

Substituting into equation (2) to solve for v_{bfp}:

$$Mv_{ci}\sin(\theta+\varphi) = M(v_{bfp} + \frac{mv_{bfp}r^2}{I}) + mv_{bfp}$$

Expanding:

$$Mv_{bfp} + mv_{bfp} + M\frac{mv_{bfp}r^2}{I} = Mv_{ci}\sin(\theta+\varphi)$$

Factorizing and dividing:

$$v_{bfp} = \frac{Mv_{ci}\sin(\theta+\varphi)}{M + m + \frac{Mmr^2}{I}}$$

Dividing by M:

$$v_{bfp} = \frac{v_{ci}\sin(\theta+\varphi)}{1 + \frac{m}{M} + \frac{mr^2}{I}} \quad (9)$$

Now that both v_{bfn} and v_{bfp} can be calculated, the spin rate can be obtained from the previous rearrangement of equation (3):

$$\omega_{bf} = \frac{-mv_{bfp}r}{I} \quad (10)$$

The launch angle can be determined using trigonometry:

$$\psi = \theta - tan^{-1}(\frac{v_{bfp}}{v_{bfn}}) \quad (11)$$

The ball speed can be determined by the vector addition of its components:

$$v_{bf} = \sqrt{v_{bfn}{}^2 + v_{bfp}{}^2} \quad (12)$$

Theoretical relationships

Only the final coefficients will be rounded to 4 s.f. to preserve precision and all relationships will be presented over $0° < \theta < 90°$ - dynamic lofts which are physically possible. The following values substituted used to obtain theoretical relationships:

- $M = 0.286\ kg$, the mass of a pitching wedge (PW) clubhead
- $v_{ci} = 82\ mph = 36.6573\ ms^{-1}$, my average for PW
- $\varphi = 0°$ so that a definite relationship with dynamic loft can be obtained

For spin rate, substituting equation (4) for I and (9) for v_{bfp} into equation (10), substituting the above values and simplifying provides the relationship:

$$\omega_{bf}\ [rpm] = Spin\ rate\ [rpm] = 11120 \sin(\theta)$$

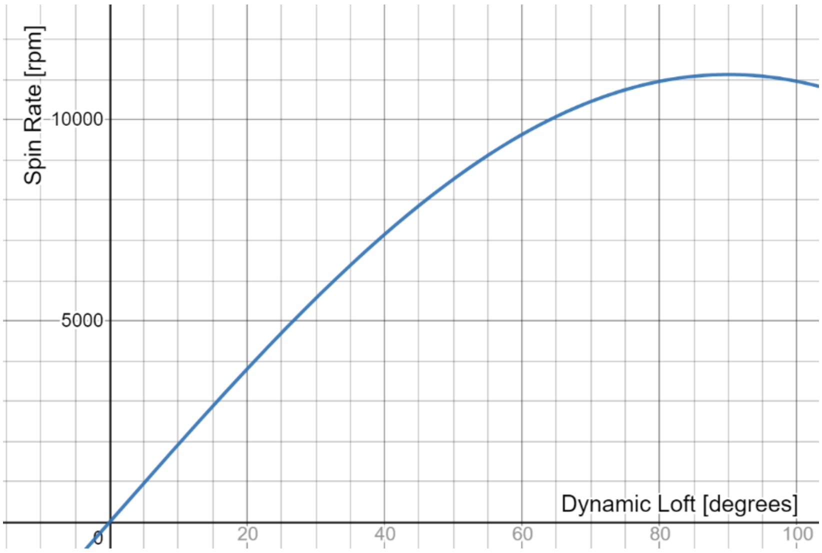


Figure 5. Theoretical Relationship between Dynamic Loft and Spin Rate, drawn with *Desmos* (2011)

For launch angle, substituting equation (9) for v_{bfp} and (8) for v_{bfn} into equation (11), substituting the above values and simplifying provides the relationship:

$$\psi\ [degrees] = Launch\ angle\ [degrees] = \theta - tan^{-1}\left(\frac{2.321 \tan(\theta)}{13.62 - 0.7783 \cos(\theta)}\right)$$

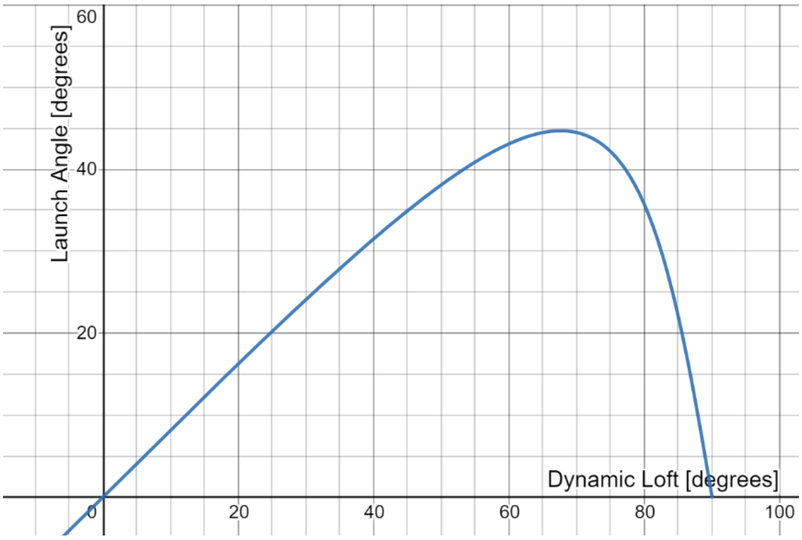


Figure 6. Theoretical Relationship between dynamic loft and launch angle, drawn with *Desmos* (2011)

For ball speed, substituting equation (9) for v_{bfp}, (8) for v_{bfn} and (6) for e into equation (12), substituting the above values and simplifying provides the relationship:

$$v_{bf}\ [ms^{-1}] = Ball\ speed\ [ms^{-1}]$$
$$= \sqrt{3452\ cos^2(\theta) - 394.6\ cos^3(\theta) + 11.28\ cos^4(\theta) + 100.3\ sin^2(\theta)}$$

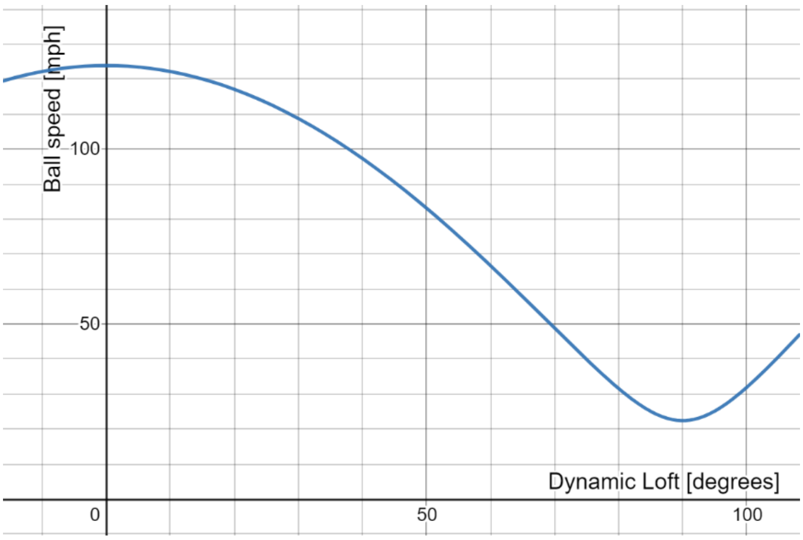


Figure 7. Theoretical Relationship between dynamic loft and ball speed, drawn with *Desmos* (2011)

Experimental data collection

Due to the assumptions of the theoretical model, it is likely that the theoretical relationships are not entirely accurate, so comparing and evaluating against experimental data would better answer the research question.

Methodology

The Trackman 4 launch monitor was set up according to instructions. TaylorMade P770 irons and Srixon yellow range golf balls were used. Golf shots were hit by me because I did not have access to advanced robotics equipment used to mimic golf swings. Resultantly, human errors involved in the golf shots were unavoidable.

To change the independent variable, clubs from PW to 4 iron were used to obtain 7 different values of roughly equal intervals for dynamic loft over a sufficiently wide range. Clubs of lower or higher lofts were not used because they are different types of clubs; their significantly different shape, mass, clubface pattern, and length could disrupt trends in the data.

To minimise the effect of human errors, I hit on average 10 shots with each club until I had a minimum of 4 shots with concordant club speed, dynamic loft and attack angle, a reasonably small impact offset, spin axis, launch direction, club path, and face angle. These data are then averaged to minimise the effect of any inconsistencies.

Figure 8b. P770 irons (front view)

Figure 8a. P770 irons (top view)

Figure 8c. Srixon yellow range golf ball

Controlled variables

The following factors were controlled to the best of my abilities by attempting to hit every shot the same way.

- Clubhead speed: 82 mph – my average
- Attack angle: -2°
- Club path: 0°
- Face angle: 0°
- Impact offset: (0, 0)

Results and comparison

Table 1. Raw data table of key variables (shots that passed the selection criterions)

Club	Index	Dynamic Loft [degrees] (±0.1)	Spin Rate [rpm] (±1)	Launch Angle [degrees] (±0.1)	Ball Speed [mph] (±0.1)
4 iron	1	12.8	3875	9.5	118.0
	2	12.7	4142	9.3	121.1
	3	13.1	4123	9.6	119.8
	4	12.5	4094	9.1	120.0
5 iron	1	17.5	5198	13.1	119.6
	2	14.9	4819	10.7	117.7
	3	14.8	5057	10.5	118.7
	4	16.6	5100	12.1	114.0
	5	17.4	5393	12.8	117.7
	6	15.3	5077	11.0	117.0
6 iron	1	18.4	5949	13.3	118.2
	2	19.7	5564	14.9	117.4
	3	18.5	5712	13.6	116.3
	4	18.6	5558	13.8	116.6
	5	19.0	5713	14.1	116.9
7 iron	1	21.9	6248	16.5	114.9
	2	22.8	6677	16.8	112.2
	3	21.3	6653	15.5	115.8
	4	23.1	6708	17.3	114.9
	5	21.2	6268	15.9	117.0
8 iron	1	24.0	7749	17.0	111.9
	2	25.3	7240	18.7	111.1
	3	25.9	7477	19.1	110.6
	4	25.9	7809	18.8	111.2
9 iron	1	27.6	8351	19.7	106.4
	2	28.5	8260	20.6	105.5
	3	28.0	8378	20.1	107.4
	4	29.2	8449	21.3	107.8
PW	1	81.2	9184	23.4	103.1
	2	82.4	9099	21.2	99.3
	3	82.0	9352	23.6	101.1
	4	81.5	9106	23.0	101.5
	5	83.0	9397	22.4	100.5
	6	82.5	9242	23.4	103.2
	7	81.6	9389	24.2	101.4
	8	81.0	9378	24.9	100.9

The uncertainties shown in **Table 1** are ± the most precise digit displayed by Trackman, the best indication of its precision.

Table 2. Processed data table of key variables

Club	Avg. Dynamic loft [degrees]	Avg. Spin Rate [rpm]	Avg. Launch Angle [degrees]	Avg. Ball Speed [mph]
4 iron	12.78 ± 0.40	4059 ± 130	9.38 ± 0.35	119.7 ± 1.7
5 iron	16.1 ± 1.5	5107 ± 290	11.7 ± 1.4	117.4 ± 2.9
6 iron	18.80 ± 0.75	5699 ± 200	13.90 ± 0.90	116.9 ± 1.1
7 iron	22.1 ± 1.1	6513 ± 230	16.4 ± 1.0	115.0 ± 2.5
5 iron	25.3 ± 1.1	7569 ± 290	18.4 ± 1.2	111.20 ± 0.80
9 iron	28.33 ± 0.90	8630 ± 100	20.43 ± 0.90	106.8 ± 1.3
PW	32.5 ± 1.1	9268 ± 150	23.3 ± 2.0	101.4 ± 2.1

The total uncertainty in each parameter in **Table 2** is calculated using the half-range rule.

Table 3. Averages for other controlled variables

Club	Clubhead speed [mph]	Attack Angle [deg]	Club path [deg]	Face angle [deg]	Impact offset [mm]	Spin axis [deg]	Launch direction [deg]
4 iron	82.8	-1.3	-0.2	0.3	(3, -3)	0.3	0.2
5 iron	81.5	-1.8	0.2	0.3	(2, -4)	-0.2	0.3
6 iron	81.9	-1.2	-0.5	-0.7	(2, -6)	0.5	-0.7
7 iron	82.2	-1.9	0.4	-1.1	(-5, -4)	-3.3	-0.7
8 iron	82.4	-2.1	0.8	0.4	(-3, -6)	0.3	0.5
9 iron	81.9	-2.7	-0.5	0.3	(-6, -4)	0.2	0.0
PW	82	-2.5	0.3	-0.2	(-3, 0)	-0.5	0.0

I consulted Golf Warehouse – a golf retailer – to help me measure the mass of each individual P770 clubhead because I could not do so myself.

Table 4. Mass of P770 iron clubheads measured by Golf Warehouse

Club	4i	5i	6i	7i	8i	9i	PW
Mass [kg]	0.2451	0.2498	0.2581	0.2643	0.2705	0.281	0.2865

To compare the theoretical model to experimental data, the average dynamic loft, club speed, attack angle and mass for each club were substituted into the model. The calculated spin rate, launch angle, and ball speed were in SI units, so they were converted into revolutions per minute, degrees, and miles per hour respectively because these are commonly used in golf. The results are presented graphically in **Figures 9**, **10** and **11**.

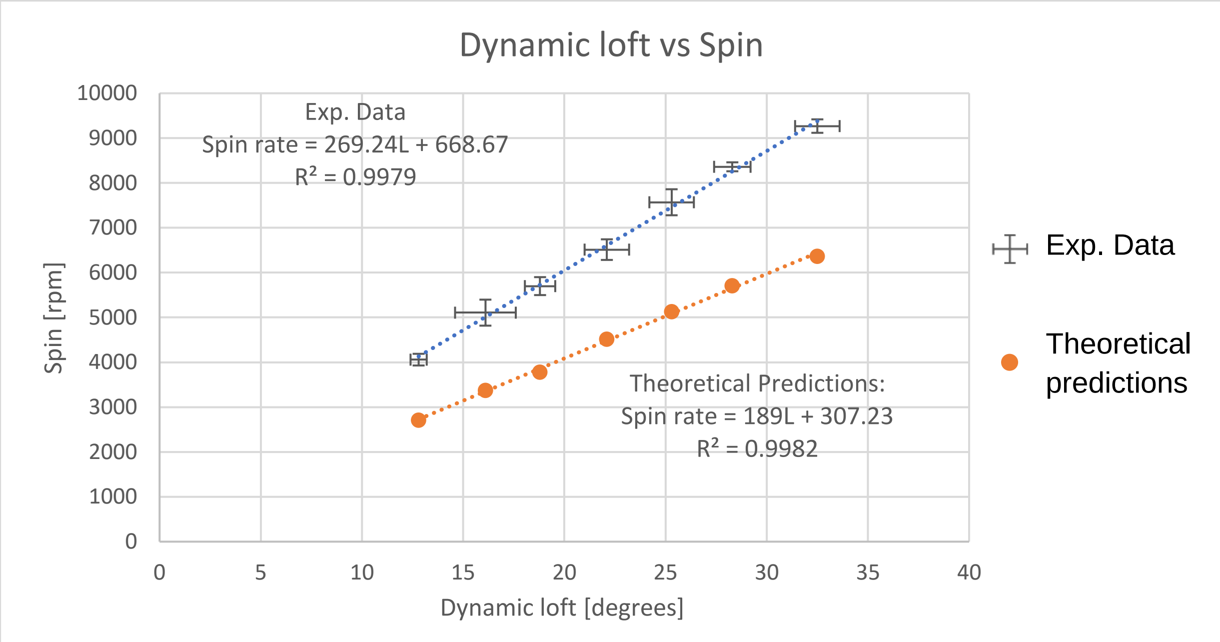


Figure 9. Graph of dynamic loft against spin rate

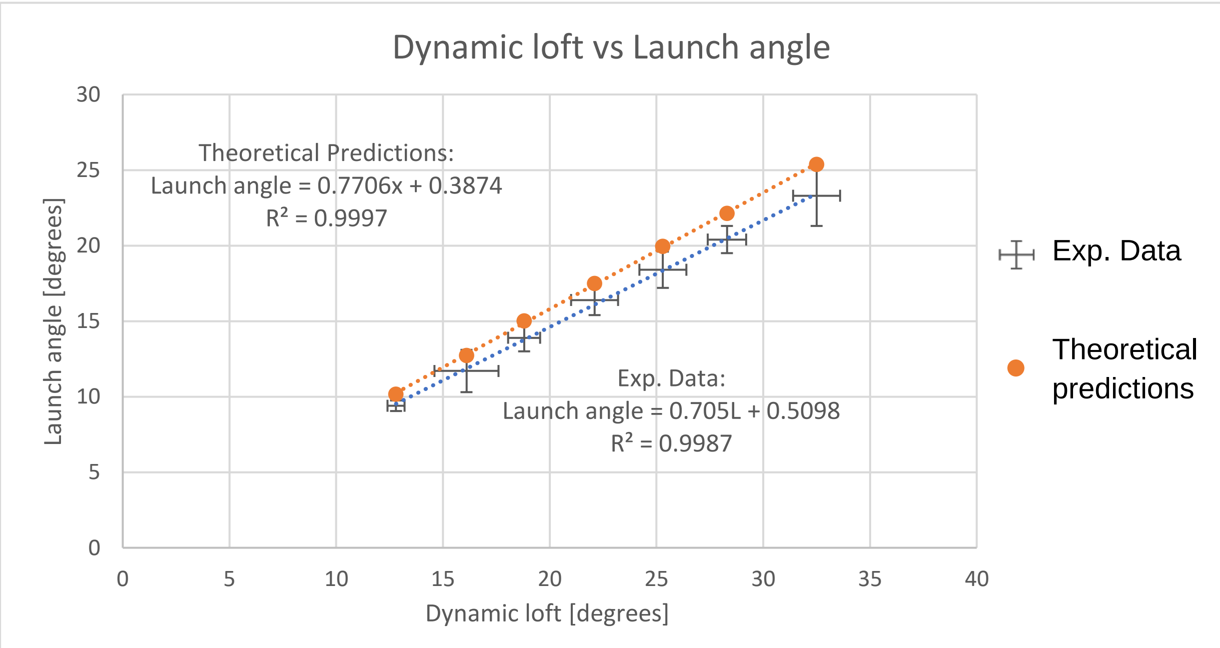


Figure 10. Graph of dynamic loft against launch angle

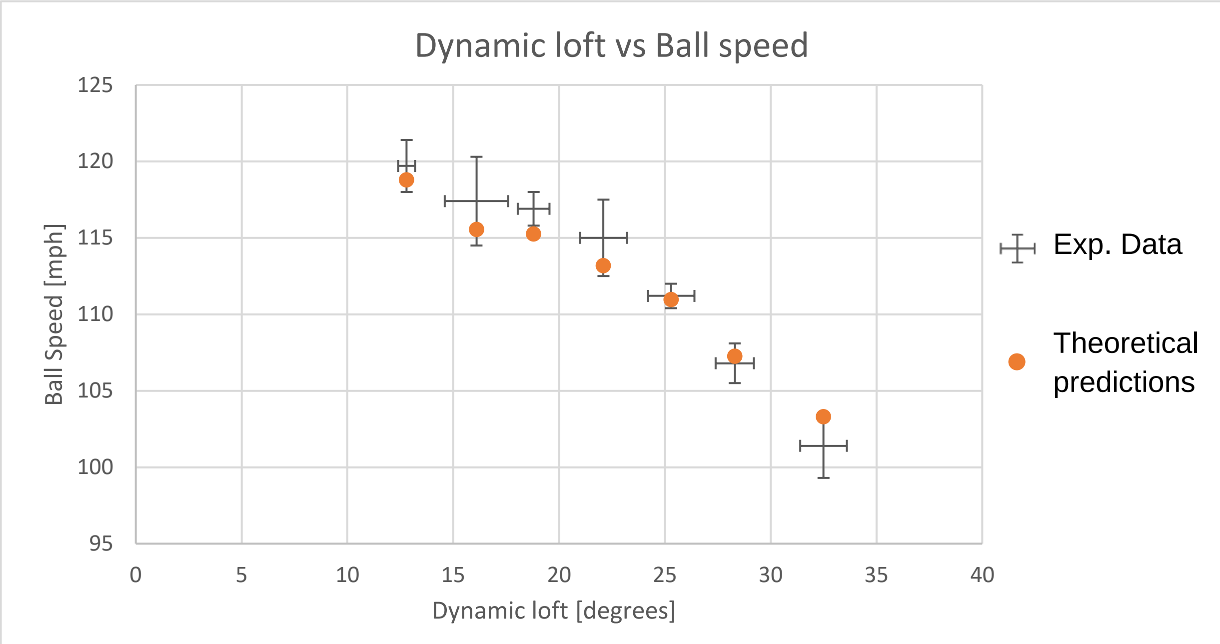


Figure 11. Graph of dynamic loft against ball speed

Trends in **Figures 9** and **10** appeared approximately linear, so linear fits were applied, however, this was not the case for **Figure 11**.

Conclusion

Each relationship in the research question will be discussed separately.

Spin rate:

The theoretical relationship in **Figure 5** suggested a sine relationship between dynamic loft and spin rate, assuming an attack angle of zero:

$$Spin\,rate\ [rpm] = 11120 \sin{(\theta)}$$

Experimental data, obtained for dynamic lofts of 12.8° to 32.5°, indicated a very strong positive linear relationship (**Figure 9**).

$$Spin\,rate\ [rpm] = 269.24L + 668.67$$

This is expected from a sine relationship, since $\sin(\theta)$ is approximately linear with $\theta < 30°$. The theoretical predictions also appeared to be linear, however, consistently underpredicted spin rate by a factor of approximately two-thirds. Despite this discrepancy, the R^2 value of 0.9979 of the experimental data suggests a strong correlation between dynamic loft and spin rate. Therefore, the data does not disagree with the theoretical model that there is a sine relationship between dynamic loft and spin rate. It can be concluded that for low dynamic lofts, this relationship can be approximated to be linear.

Launch angle:

The theoretical relationship in **Figure 6** showed a more complex trigonometric relationship between dynamic loft and launch angle:

$$Launch\ angle\ [degrees] = \ \theta - tan^{-1}(\frac{2.321\tan(\theta)}{13.62 - 0.7783\cos(\theta)})$$

Again, experimental data suggested a very strong positive linear relationship with a high R^2 value of 0.9987 (**Figure 10**):

$$Launch\ angle\ [degrees] = \ 0.705L\ +\ 0.5098$$

This is also expected because the section of **Figure 6** with $\theta < 35°$ is approximately linear. The theoretical prediction is also concordant with the experimental relationship, only being slightly higher than the experimental values. It can therefore be concluded that the relationship between dynamic loft and launch angle can be considered linear for low dynamic lofts while the true relationship is far more complex.

Ball speed:

The theoretical relationship in **Figure 7** showed that there is a complex trigonometric relationship between dynamic loft and ball speed:

$$Ball\ speed\ [ms^{-1}] = \sqrt{3452\ cos^2(\theta) - 394.6\ cos^3(\theta) + 11.28\ cos^4(\theta) + 100.3\ sin^2(\theta)}$$

Experimental data, seen in **Figure 11**, suggests a non-linear negative relationship as indicated by its shape, which is similar to the theoretical relationship in **Figure 7**. Therefore, it can be concluded that the above relationship is reliable.

Evaluation of the theoretical model

Figures 9, **10** and **11**, show that the theoretical model quite accurately predicted the relationship for launch angle and ball speed but significantly underpredicted spin rate. The theoretical predictions for launch angle and spin rate mostly lied within the error bars of experimental values. The theoretical relationships and experimental data agree with existing research (Penner, 2001; Dewhurst, 2015) in the shape of each respective relationship; spin rate and launch angle increase roughly linearly at low lofts and ball speed decreases in a downward curve. This suggests some accuracy within both the theoretical model and experimental data.

A strength of this investigation is that assumption 5 was largely satisfied as **Table 3** shows that the club path, face angle, and impact offset were all relatively small, with the exception that almost all shots had a negative y impact offset due to my swing tendencies. The golf balls used in data collection – Srixon yellow range – are 1-piece, homogenous balls, again satisfying theoretical assumptions. Experimental and theoretical conditions are mostly concordant; hence, the experimental data is likely reliable.

Although the discrepancy between theoretical predictions and experimental data for ball speed was likely due to random errors in the experimental data, as suggested by the error bars, the theoretical predictions showed a shallower curve, suggesting weaknesses in the theoretical model.

The same is true for the discrepancy in launch angle, where theoretical predictions were consistently higher than the experimental data. In the theoretical model, launch angle was dictated by:

$$\psi = \theta - tan^{-1}(\frac{v_{bfp}}{v_{bfn}})$$

This discrepancy, therefore, suggests that the ratio $\frac{v_{bfp}}{v_{bfn}}$ was underpredicted, meaning that either v_{bfp} was underpredicted or that v_{bfn} was overpredicted or a combination of both, although the extent is unclear. However, the lower experimental launch angles could also be the result of my tendency for negative y impact offsets, which introduces a torque acting clockwise on the clubhead upon impact, effectively reducing the loft by rotation and resulting in a lower launch angle. It is also common knowledge in golf that these ‘thinned’ shots launch lower.

However, the difference between the experimental data and theoretical predictions for spin rate is too significant to be explained by random errors.

The systematic error caused by my negative y impact offset tendency likely also contributed to the higher experimental spin rates. As discussed, this causes the clubhead to rotate clockwise, resulting in angular momentum in that direction. Hence, by the conservation of angular momentum, the ball will have more angular momentum in the anticlockwise direction and therefore more spin. The clockwise rotation of the clubhead will also impart more spin on the ball by gear effect (Cross, 2021). However, this is unlikely to be the only cause for such a big difference because these impact offsets were not very significant, suggesting further weaknesses in the theoretical model.

In the theoretical model, spin rate is dictated by:

$$\omega_{bf} = \frac{-mv_{bfp}r}{I}$$

It is proportional to v_{bfp}, indicating that v_{bfp} is severely underpredicted and is likely the cause for the higher theoretical launch angles rather than the overprediction of v_{bfn}, since the theoretical spin rate deviated more from experimental results than the launch angle. These observations suggest that one or more assumptions used in deriving v_{bfp}

or ω_{bf} was erroneous. It is likely to be a combination of both because any source(s) of error for v_{bfp} only caused a slight discrepancy in the prediction of launch angle. Upon reflection, I realised that equation (3),

$$I\omega_{bf} + mv_{bfp}r = 0$$

which described the conservation of angular momentum and is used in the calculation of both v_{bfp} and ω_{bf}, is most likely responsible because the initial angular momentum of the clubhead-ball system is not zero. Although the clubhead moves with linear motion, its velocity vector has a perpendicular distance $r_{\perp}$ to the point of contact (see **Figure 12**), therefore, the system has initial angular momentum of $L = Mv_{ci}r_{\perp}$ in the anticlockwise direction (Weideman, 2021). This would give rise to more spin, which would match the experimental data.

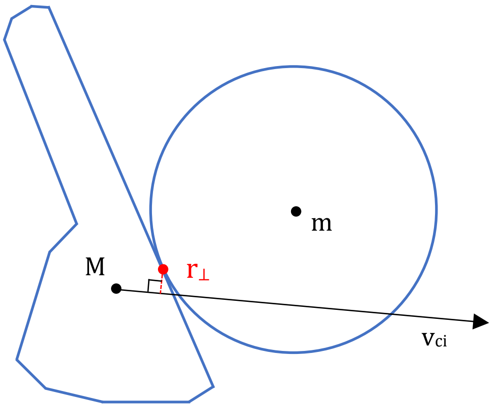


Figure 12. Side view of the clubhead-ball impact demonstrating $r_{\perp}$

It is also very likely that the oversimplification of considering the ball as a rigid object greatly contributed to this underprediction. Another study by Cross (2021) showed that the clubhead-ball impact cannot be described as 'sliding and rolling'. Instead, the deformation of the contact region allows the ball to grip the clubface as it comes to rest. As a result, the ball vibrates in the direction parallel to the clubface which causes friction force to reverse direction during impact, potentially multiple times (**Figure 13**). This means that the ball can spin faster than allowed by the rolling condition. Even greater spin is imparted on the ball if this vibration enables the normal force to act in a line below the centre of mass of the ball. USGA's (2006) report also supports this suspicion.

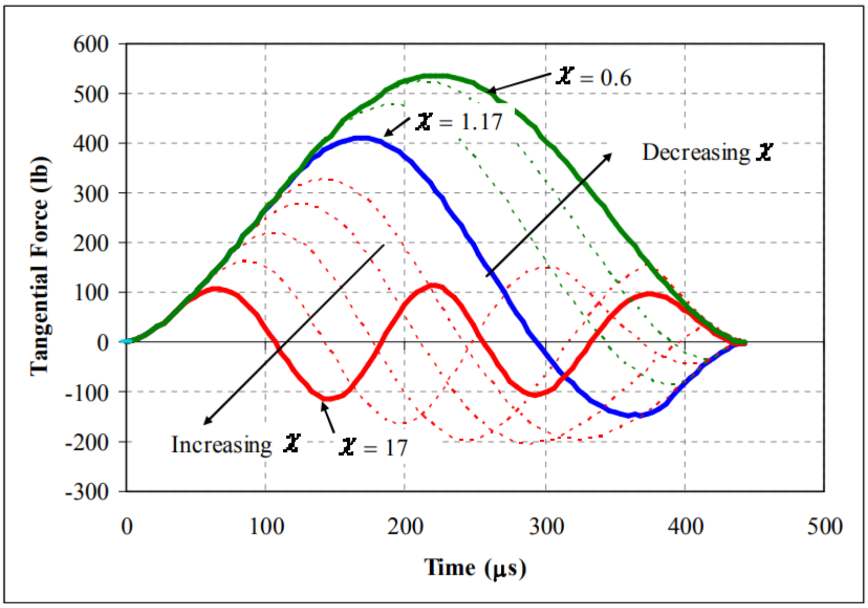


Figure 13. Effect of tangential stiffness on tangential force history (USGA, 2006)

Other possible weaknesses lie in assumptions 6 (The centre of mass of the clubhead lies on the line connecting that of the ball and the centre point of impact) and 7 (Any effects of the dimples are negligible) as they are approximations. However, they are unlikely to have significant impacts. All previous studies have ignored the effects of dimples yet some managed to produce very accurate results. Assumption 6 could explain higher experimental spin rates only if the true centre of mass of the club was higher than predicted, which will have the same effects as my negative y impact offset tendency. Unfortunately, there is no way of confirming or denying this possibility.

Discussion in relation to golf and extensions

Although this investigation focused on irons, the theoretical model can be easily adapted to find the relationships for drivers and wedges by changing the clubhead mass, where similar relationships with different coefficients will be obtained. In practice, the findings of this investigation show that:

- The spin rate increases approximately linearly with dynamic loft at low lofts, but this rate of increase slows down at higher dynamic lofts. This agrees with general knowledge in golf, as golfers increase spin by adding loft
- The launch angle increases similarly to spin rate but peaks at the dynamic loft of approximately 67°. However, this is almost never reached in golf, so generally, increasing dynamic loft will increase the launch angle, also agreeing with golf knowledge.
- Increasing dynamic loft reduces the ball speed becomes more dramatic at higher lofts. This again agrees with common golf knowledge in that higher lofts lead to lower ball speeds when the clubhead speed is identical.

Hence, increasing the dynamic loft will increase spin, launch and reduce ball speed, all of which will cause the ball to stop quickly after landing and vice versa. These findings support what I have been previously taught by coaches: that I should add loft to stop the ball quickly on the green as it results in higher shots with more spin.

Extensions could attempt to quantify how each assumption led to the inaccuracy of the theoretical model to gain a better understanding of the impact behaviour by comparing this investigation's results with more accurate models such as the one from Maw et al. (1976) along with other empirical evidence. USGA (2006) also discovered that at higher lofts, the friction coefficient between the clubface and the ball significantly affects the dynamic loft-spin rate profile (**Figure 14**). This could also lead to an investigation into the kinematics and elastic behaviour of the impact at higher dynamic lofts since this investigation assumed an infinite friction coefficient by assuming that the ball enters rolling motion. Further, how the dynamic loft-spin rate, launch angle and ball speed profiles vary with mechanical properties of the clubhead and ball can also be investigated to uncover these relationships for wedges and drivers.

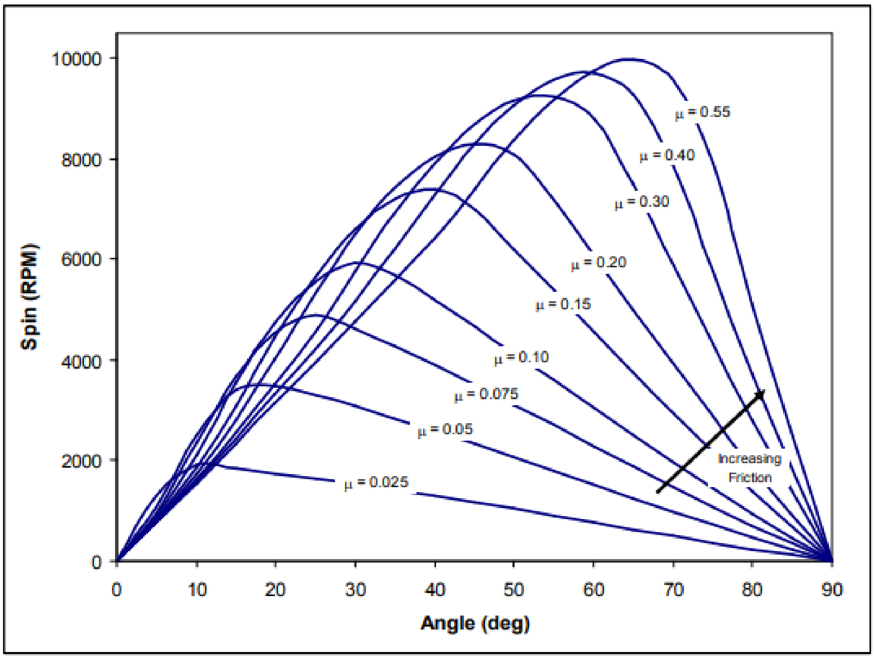


Figure 14. Effect of coefficient of friction on spin rate (USGA, 2006)

Bibliography

Cermik, O., Ghaednia, H., & Marghitu, D. B. (2017). Analytical Study of the Oblique Impact of an Elastic Sphere with a Rigid Flat. *Springer Proceedings in Physics*, 33–40. https://doi.org/10.1007/978-3-319-69823-6_4

Chou, P. C., Liang, D., & Yang, J. (1994). Contact forces, coefficient of restitution, and spin rate of golf ball contact. In A. J. Cochran & M. Farrally (Eds.), *Science and Golf II: Proceedings of the World Scientific Congress of Golf* (1st ed., p. Chapter 45). Taylor & Francis.

Cochran, A., & Stobbs, J. (1968). *The Search for the Perfect Swing* (1st ed.). J. B. Lippincott Company.

Cross, R. (2002). Grip-slip behavior of a bouncing ball. *American Journal of Physics*, *70*(11), 1093–1102. https://doi.org/10.1119/1.1507792

Cross, R. (2021). Experimental study of the gear effect. *European Journal of Physics*, *42*(6), 065013. https://doi.org/10.1088/1361-6404/ac2938

Desmos. (2011). [Graphing Calculator]. Eli Luberoff. https://www.desmos.com/calculator

Dewhurst, P. (2015). *The Science of the Perfect Swing* (Illustrated ed.). Oxford University Press. https://books.google.co.nz/books/about/The_Science_of_the_Perfect_Swing.html?id=oRo3rgEACAAJ&redir_esc=y

Down The Middle. (2020, October 24). *Bryson DeChambeau high-speed video of his golf ball at impact* [Video]. YouTube. https://www.youtube.com/watch?v=c-0dDKfw4NI

Hahn, C. (2021, January 8). 40+ TrackMan Parameters. TrackMan Golf. Retrieved April 25, 2022, from https://blog.trackmangolf.com/40-trackman-parameters/

Haron, A., & Ismail, K. A. (2012, September). Coefficient of restitution of sports balls: A normal drop test. *IOP Conference Series: Materials Science and Engineering*, 012038. https://doi.org/10.1088/1757-899x/36/1/012038

Iwatsubo, T., Kawamura, S., Miyamoto, K., & Yamaguchi, T. (1998). Analysis of Golf Impact Phenomenon and Ball Trajectory. *JSME International Journal Series C*, *41*(4), 822–828. https://doi.org/10.1299/jsmec.41.822

Keatley, R. K. (2021, August). *Fore! Analyzing the Performance of a Golf Ball with Simulation*. COMSOL Multiphysics. https://www.comsol.com/blogs/fore-analyzing-the-performance-of-a-golf-ball-with-simulation/

Lieberman, B. B., & Johnson, S. H. (1994). An analytical model for ball-barrier impact. In A. J. Cochran & M. Farrally (Eds.), *Science and Golf II: Proceedings of the World Scientific Congress of Golf* (1st ed., p. Chapter 48). Taylor & Francis.

Mase, T. M. (2004). *Experimental Benchmarking Golf Ball Mechanical Properties*. Proceedings of the Society of Experimental Mechanics X International Congress & Exposition on Experimental and Applied Mechanics. https://content-calpoly-edu.s3.amazonaws.com/me/1/documents/tom-mase/sem04ballpaper.pdf

Maw, N., Barber, J., & Fawcett, J. (1976). The oblique impact of elastic spheres. *Wear*, *38*(1), 101–114. https://doi.org/10.1016/0043-1648(76)90201-5

Penner, A. R. (2001). The physics of golf: The optimum loft of a driver. *American Journal of Physics*, *69*(5), 563–568. https://doi.org/10.1119/1.1344164

Shutterstock. (n.d.). *Golf Ball* [Image]. Collins Dictionary. https://www.collinsdictionary.com/images/full/golfball_170415218_1000.jpg?version=4.0.219

TaylorMade Golf. (n.d.). *P770 Irons* [Photograph]. P770 IRONS. https://www.taylormadegolf.eu/P770-Irons/DW-TA155.html?lang=en_DE

United States Golf Association. (2006, August). *Interim Report: Study of Spin Generation*. USGA. https://www.randa.org/~/media/files/equipmentsubmissions/interim-report-on-study-of-spin-generation.ashx

USGA & R & A. (2019, January). *The Equipment Rules*. United States Golf Association. https://www.usga.org/content/dam/usga/pdf/Equipment/Equipment%20Rules%20Final.pdf

Weideman, T. (2021, November 18). 6.1: Linking Linear and Angular Momentum. Physics LibreTexts. Retrieved May 26, 2022, from https://phys.libretexts.org/Courses/University_of_California_Davis/UCD%3A_Physics_9A__Classical_Mechanics/6%3A_Angular_Momentum/6.1%3A_Linking_Linear_and_Angular_Momentum